Mechanics Classwork 1, 22/10/10, 10:00: Jogger Preservation

Introduction and Aims
This classwork deals with an idealised kinematics problem. Although the physics is essentially rather straightforward the implementation of the mathematics in the problem is intended to stretch you. In particular the classwork aims to improve your skills in the application of curve sketching and differential calculus in physical problems.

The Problem
A jogger is running along the middle of a road with constant velocity $v$ (relative to the road). At time $t = 0$ the jogger notices a truck of width $2b$, travelling along the middle of the road a distance $L$ behind him, and heading towards him with velocity $w$ (relative to the road) where $w > v$. The situation is shown in the schematic figure below:

The jogger problem: In this problem the jogger (the dot) runs at speed $v$ at angle $\theta$ to the truck’s direction. For what angle are the jogger’s chances of survival greatest?

The jogger decides to get clear of the truck and makes an instantaneous(!) direction change, at angle $\theta$ to the truck’s direction, to get off the road. Assuming the jogger moves in a straight line following the initial direction change, and maintains a constant speed $v$ at all times, the principal problem is to determine the best direction for the jogger to run in to preserve himself. At first thought it may seem that running straight off the road (perpendicular to the truck) may be the best option. But running at an angle means the jogger is moving away from the truck so maybe a slight angle is the best option. The questions go through the problem in stages.

Questions
1. In the first part of the problem you will consider how much time passes before the truck hits the jogger:
a) Imagine the jogger misjudges the situation and thinks he can outrun the truck i.e. $\theta = 0$. Find an expression for the time $t$ when the truck catches up with the jogger.

b) Now imagine the jogger has a case of blind panic and runs straight at the truck! ($\theta = \pi$). Find an expression for the time until impact in this case.

c) Find an expression for the time if the jogger has more presence of mind and tries to run straight off the road in the minimum distance possible ($\theta = \pi/2$). Assume the jogger isn’t fast enough to escape the truck in this case!

d) Generalise the equation to write an expression for the time of impact as a function of general angle $\theta$. Verify this agrees with the first three answers and sketch a rough graph of $t$ vs. $\theta$ from $0 – \pi$.

2. Next, work how much “vertical” (i.e. perpendicular to the original direction) distance the jogger covers, $y$, before impact:

   a) Write down an expression for $y$ in terms of $t$, $v$ and $\theta$
   
   b) Substitute in the value of $t$ from part 1. to show that

   $$y = \frac{vL}{w - v \cos \theta}$$

   Sketch a rough graph of $y$ vs. $\theta$ from $0 – \pi$ (this is tricky – you may use a calculator to compute values) and convince yourself that the curve contains a maximum. What significance does this maximum have for the jogger?

3. Finally work out what the jogger’s chances of survival:

   a) Find the value of $\theta$ for which $y$ is a maximum and substitute this back into the equation from 2.b) to show that the maximum value of $y$ is given by

   $$y_{\text{max}} = \left(\frac{v}{w}\right)L \left(1 - \left(\frac{v}{w}\right)^2\right)^{-1/2}$$

   b) If the jogger moves at 10 ms$^{-1}$ (Usain Bolt speed!) and the truck at 25 ms$^{-1}$ (below the National Speed Limit) compute the jogger’s optimum angle. If the initial separation, $L = 3$m, and the truck’s width 2.5 m will the jogger survive if he runs i) straight off the road ($\theta = \pi/2$) and ii) at the optimum angle?

4. This problem has made a number of unrealistic assumptions. Discuss these assumptions and how the calculation may potentially be modified to make the problem more realistic. Detailed calculations are not required.
Mechanics Classwork 2, 4/11/10, 15:00: Gravitational Field Strength

Introduction and Aims
This classwork studies the variation of the gravitational field strength with distance from the Earth’s surface. The main aims are to improve your understanding of gravitational fields near a planet’s surface, and to illustrate how the binomial theorem and integral calculus can be used to solve problems in physics.

Notes
In this classwork take \( G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2} \), the mass of the Earth, \( M_E = 5.97 \times 10^{24} \text{ kg} \) and the radius of the Earth to be \( R_E = 6.37 \times 10^6 \text{ m} \)

Questions
1. Consider an object that is raised from the surface of the Earth to a height \( h \) above the surface:
   a) Write down formulae for the gravitational field strength at i) the surface and ii) height \( h \). State any assumptions these formulae make.
   b) Input numbers to work out the change in field strength, \( \Delta g(h=1\text{m}) \), on moving from the surface to a height of 1 m. Comment on your answer.
   c) Use your answers for part a) and the binomial theorem to show that if \( h \ll R_E \) then

\[
\Delta g \approx \frac{2GM_E h}{R_E^3}
\]

Input numbers to show this agrees with the answer in part b) and sketch a plot of the magnitude of \( g \) as a function of height above surface for low height.
2. Now consider an object, mass $m$, that is released from rest from a great height $z_0$ from the Earth’s centre in the absence of any other masses. Assume that $M_E \gg m$.

a) Describe in words what will happen to the object

b) Apply Newton’s 2nd Law to the object to write down a differential equation for the velocity of the object as a function of its position.

c) Solve the equation to show that the velocity of the particle towards the Earth when it has fallen to a distance $z$ from Earth’s centre can be written

$$v(z) = \sqrt{2GM_E \left( \frac{1}{z} - \frac{1}{z_0} \right)^{1/2}}$$

How could this equality have been obtained using the conservation of energy?

d) Revert back to the quantities in part 1 and rewrite the equation above in terms of $R_E$ and $h$ instead of $z_0$ and $z$. Show that if the particle falls to Earth when $h \ll R_E$ then the expression for the velocity as a function of distance is equivalent to that given by the equations for constant acceleration. (NB it is possible to invoke the binomial theorem again for this part but it is not essential.)

e) Taking the other extreme if the Earth were actually a point mass(!), what would be the final value of a velocity of a falling object? Does this make sense? What physics would have to be used to solve the problem properly?

f) It is possible to further integrate the velocity equation further to compute the time taken for the particle to fall between $z$ and $z_0$. This leads to a rather tricky integral (try it – but not now - if you fancy a challenge; see McCall, section 2.12) and the solution is rather ungainly:

$$\frac{\pi}{2} - \sin^{-1} \left( \frac{z}{z_0} - \frac{1}{2} \sin \left[ 2 \sin^{-1} \left( \frac{z}{z_0} \right) \right] \right) = \frac{2GM_E}{z_0^3} \cdot t$$

Again, considering that the Earth is a point mass, find an expression for the total fall time. Use this to provide an order of magnitude estimate of how long it would take two rocks 1 m apart in space to collide starting from rest.

**NB** McCall discusses some further properties of the solution to the equation (beyond the scope of this course) in his paper “Gravitational Orbits in One Dimension”, American Journal of Physics, 74, 1115 – 1119 (2006) if you wish to read up on this further.
Introduction and Aims
This classwork aims to improve your understanding of pseudo forces in linearly accelerating reference frames and rotating systems. Even though the material is not too tricky questions of this nature will not feature on the exam or any other assessed work. It will, however, enhance your understanding of Classical Mechanics and provides a theoretical basis for understanding some fascinating and important phenomena.

Questions
1. Two astronauts are standing 10 m apart in a spaceship accelerating upward at $a_y = 3 \text{ ms}^{-2}$. One throws a ball to the other parallel to the floor they are standing on. If the ball is released at a height of 2 m, what initial speed does it need to reach the other astronaut before hitting the floor? Solve the problem from the perspective of both (a) a noninertial observer (inside the ship) and (b) an inertial observer (outside the ship). Sketch graphs (on the same axes) of the vertical position of (1) the ball and (2) the floor of the rocket as a function of time as seen by the inertial observer. Repeat for the non-inertial observer.
2. This question concerns the Coriolis acceleration. Part (a) gets you to derive a simplified version of the equation of the equation for the Coriolis acceleration seen in lectures and part (b) studies a hypothetical, though measurable, real world Coriolis acceleration effect.

a) Consider a circular platform (frame $S'$) of radius $R$ rotating with angular velocity $\omega$ relative to an inertial frame $S$. At time $t = 0$ a person at the centre $0$ throws a ball at speed $v'$ towards a person standing on the rim of the platform.

i) Write down an expression for the time taken for the ball to reach the rim

ii) Where will the person on the rim have moved to by this time? If the ball’s speed is large so that $v' \gg R\omega$ and the arc the person has moved round approximates a straight line, write down an expression for this straight line distance.

iii) If the acceleration of the ball relative to the person on the rim is given by $a'$ and is constant go on to show that this Coriolis acceleration is given by

$$a' = 2\omega v$$

In which direction does the person on the rim perceive the acceleration to be?

iv) The magnitude of the Coriolis force on object moving in a rotating frame is hence given by $F_{\text{Cor}} = 2m\omega v'$. Explain why it is referred to as a fictitious force.

b) Consider a snooker table on the North pole(!). When playing a straight shot, what direction would the Coriolis force cause a ball to swerve in? If a shot is played across a table of length $L$, at velocity $v$ show that the deviation due to the Coriolis effect is given by

$$s = \frac{2\pi L^2}{Tv}$$

where $T = 1$ day. Note that this is inversely proportional to velocity, whereas the size of the force is directly proportional (why?). Would a snooker player be able to see any noticeable deviation from his shot?
Mechanics Classwork 4, 23/11/10, 15:00: The Sliding Ladder Problem

Introduction and Aims

The problem of the ladder sliding down a wall is one of the vintage problems in Classical Mechanics. Most mechanics textbooks will contain some version or other of the problem, and indeed it appeared as a Comprehensive Exam question here in 2008.

This version involves a ladder sliding under its own weight against frictionless surfaces. Variants may include friction on either surface, people standing on a light ladder, multiple masses, inclined surfaces or additional complexities like people holding the ladder at the base.

The aim is to improve your understanding of rotational motion, curve sketching, separation of motion into rotating and translational parts and the use of differential calculus in solving mechanics problems.

Notes

There is certainly more than one way to solve this problem. This Classwork takes you through the stages of one approach. If this were assessed work (an APS or an exam) you would only get credit for following the exact instructions however you are quite welcome to try alternative methods here if you wish!

The Problem

A uniform ladder of length \( l \) and mass \( m \) stands on a frictionless floor and leans against a frictionless wall. The floor and wall are perpendicular. The ladder is initially held motionless with the bottom end an infinitesimal distance from the wall. It is released, and consequently the top end slides down the wall and the bottom end slides along the floor. At some point the ladder loses contact with the wall (try this with a plastic ruler and some slippery surfaces at \( 90^\circ \)). Find the angle between ladder and wall (\( \theta_{\text{crit}} \)), and the velocity of the centre of mass (\( \vec{v} \)), at the instant the ladder loses contact:

a) Sketch a free body diagram for the ladder when it is at an angle, \( \theta \), to the wall.
b) As the ladder falls it can be considered to have a translational motion combined with a rotational motion of angular velocity $\omega$ about the centre of mass of the ladder. Use the Conservation of Energy to show that

$$gl(1 - \cos \theta) = v^2 + \frac{1}{12}l^2\omega^2$$

c) Verify that while the ends of the ladder remain in contact with the wall and floor that the centre of mass of the ladder moves in a quarter circle of radius $l/2$ (try manually plotting out the centre of mass location of the ladder slide if you’re not sure and it should become clear). Hence write down an expression relating $\omega$ and $v$ and go on to prove that the horizontal component of the velocity of the centre of mass of the ladder, $v_x$, is

$$v_x = \frac{1}{2}\sqrt{3gl(1 - \cos \theta)} \cos \theta$$

d) Sketch a rough plot of $v_x$ vs. $\theta$ and convince yourself there is a maximum between 0 and $\pi/2$. What does this maximum correspond to? (Hint: think about how the horizontal component of the acceleration varies with angle. Does the plot have much meaning beyond the maximum?) Go on to show that the angle the ladder leaves the wall is given by $\theta_{crit} = \cos^{-1}\frac{2}{3} \approx 48.2^\circ$ and the horizontal velocity component is $v_{x\,crit} = \frac{1}{3}\sqrt{gl}$.

e) Optional Extra: What happens to the contact force with the wall as the ladder slides? For what angle does it become a maximum? $\theta_{max\,contact\,force} = \cos^{-1}\left(\frac{1+\sqrt{19}}{6}\right) \approx 26.7^\circ$
Mechanics Classwork 5, 30/11/10, 15:00: Simple Harmonic Motion

Introduction and Aims
This classwork asks you to study the motion of systems that are displaced from their equilibrium position. The ultimate aim is for you to improve your understanding of the physics of oscillating systems; this will be done by studying the dynamics and energies of the systems.

Questions
1. A block of density \( d_b \) has a horizontal cross-sectional area \( A \) and a vertical height \( h \). It floats in a fluid of density \( d_f \). The block is pushed down and released. Assuming no energy losses show that it executes SHM with period

\[
T = 2\pi \sqrt{\frac{d_B h}{d_F g}}
\]

NB: Remember Archimedes Principle – the upthrust force on an object equals the weight of the fluid displaced. First consider the system when in equilibrium, then look at the system with the block displaced.
2. A plank of wood rests on two rotating cylinders a distance $d$ apart as shown in the figure below:

a) Given that (i) the cylinders are made of the same material, (ii) are rotating fast enough so that they permanently slip on the plank’s surface, thereby supplying a maximum frictional force, and (iii) that static friction and sliding friction, $\mu$, are the same for the cylinder/plank boundary show that the plank will oscillate back and forth with period

$$T = \pi \sqrt{\frac{d}{2\mu g}}$$

NB refer to Problem Sheet 5 Question 1 as a reminder of how to compute the contact forces on each cylinder.

b) Although kinematically the plank obeys SHM type behaviour explain why this system is not strictly undergoing SHM. **Hint:** consider the energy of the system.
Mechanics Classwork 6, 8/12/10, 10:00: Orbits

Introduction and Aims
This classwork is an old exam question (April 2007) from the Mechanics and Relativity exam. This question formed 32% of the total mark in a 2 hour exam and so should take just under 40 minutes of exam time. After each question I have added a note in bold indicating how I would be inclined to alter the question were I to write it myself.
Of course in this classwork you have your notes, friends and demonstrators to help you – exam conditions are different!

Questions
i) A planet experiences a gravitational force from a star. Assuming that this is the only force acting on the planet, show that \( L \), its angular momentum about the star, is constant.

\[ 6 \text{ marks} \]

VT possible changes: Preamble before the main question stating something like: “This question is about planetary orbits and angular momentum” to set the scene. Part (i) would read more like “Define angular momentum. Explain why a planet’s angular momentum remains constant when it orbits a star if the only force the planet experiences is the gravitational force due to the star.” 6 is maybe on the generous side.

ii) The planet follows an elliptical orbit around a star. Show that the rate at which a line joining the planet to the star sweeps out area is given by

\[ \frac{dA}{dt} = \frac{L}{2m} = \text{constant} \]

where \( L \) is the magnitude of the angular momentum and \( m \) is the mass of the planet (i.e. Kepler’s second law of planetary motion).

\[ 8 \text{ marks} \]

VT possible changes: Not many but may include a diagram.
iii) Assuming now that the planet actually follows a circular orbit of radius $R$, show that the period of its orbit is given by

$$T = \frac{2\pi R^{3/2}}{(G m)^{3/2}}$$

where $m$ is the mass of the star, and $G$ is the gravitational constant ($=6.67 \times 10^{-11}$ Nm$^2$kg$^{-2}$).

VT possible changes: Not many but (ii) and (iii) are standard “bookwork” type questions i.e. a student with a good memory could reproduce such formulae without much thought… One question of its type would feature in a long question like this but two is too many.

Also, $m$ was used for the mass of the planet in the previous part, and mass of the star here which isn’t ideal.

iv) Assuming that the Earth follows a circular orbit around the Sun (mass $1.99 \times 10^{30}$ kg) of period 1 year, calculate the distance from the Earth to the Sun

VT possible changes: 3 marks would be fine at A level but is generous for an Imperial College undergraduate course for a simple equation rearrangement and calculator work. Award 2 marks only (that in itself is a bit kind) or drop the “hint” that the period is 1 year.

v) A binary star system consists of two stars, of masses $M_1$ and $M_2$, following circular orbits, of radii $r_1$ and $r_2$, about their common centre of mass. Write down Newton’s second law for each of the stars separately, and, hence, show that the expression for the period, $T$, found in part (iii) can be used for the binary star system if $R$ is replaced by the separation of the stars’ centres and $M$ is replaced by $M_1 + M_2$.

VT possible changes: diagram would be included. 8 marks is a little kind but there is an argument for leaving it that high.

This style of having a “challenge” problem at the end of the question, is normal. They will often be “show that” questions. Because of the fewer marks earlier this would be expanded to a 10 mark part with some additional complication (e.g. what happens if the energy of the system is increased) for the extra 2.
Mechanics Classwork 7, 17/12/10, 10:00: Collisions

Introduction and Aims
This classwork follows on directly from the final lecture in the course. Both questions concern collisions between two objects; the first considering linear momentum only, yielding an important result, the second involves linear and angular momentum. Both questions have the level of difficulty of a moderate first year mechanics exam question.

Questions
1. Show that if a linear elastic collision between two snooker balls of the same mass occurs such that before the collision one of the balls is stationary, and after the collision the balls move in different directions, then the angle between the snooker balls’ velocities is $90^\circ$.
(Hint: draw a Before and After sketch for the collision, write down the variables and apply the relevant conservation laws. It is possible to solve this by resolving components but much nicer to consider the momenta in their vector formats.)

2. A particle of mass $m = 0.5 \ kg$ moving at speed $u = 4 \ m/s$ strikes a dumbbell consisting of two blocks of equal mass $M = 1 \ kg$ separated by a massless rod of length 2 m:

The dumbbell and the particle are free to slide on a horizontal surface. Find:

a) The speed of the centre of mass of the system after the particle sticks to one of the blocks ($0.8 \ m/s$)
b) The angular velocity of the system about the centre of mass ($0.667 \ rad/s$)
c) The loss in kinetic energy of the system due to the collision. What happens to this energy? ($2.67 \ J$)
Mechanics Classwork 1: Jogger Preservation

1. Moving away
   \[ L + v_t = w t \]
   \[ t = \frac{L}{w - v} \]
   Moving towards
   \[ v_t = L - wt \]
   \[ t = \frac{L}{w + v} \]

2. a) 'Vertical' component
   \[ y = v \sin \theta \]
   b) Vertical distance covered
   \[ y = v \sin \theta \times t \]
   Substitution gives
   \[ y = \frac{vL}{w} \sin \theta \]

3. a) The maximum is when
   \[ \frac{dy}{d\theta} = 0 \]
   Differentiating:
   \[ \frac{dy}{d\theta} = \frac{vL}{w} \left( \frac{w + v \cos \theta}{(w - v \cos \theta)^2} \right) \cdot \sin \theta - \frac{vL}{w} \cdot \frac{w - v \cos \theta}{(w - v \cos \theta)^2} \]
   \[ = \frac{vL}{w} \cdot \frac{w v \sin \theta + v \cos \theta}{(w - v \cos \theta)^2} \]
   \[ = \frac{vL}{w} \cdot \frac{w \cos \theta - v}{(w - v \cos \theta)^2} \]
   So when \( \frac{dy}{d\theta} = 0 \)
   \[ vL \left( \frac{w \cos \theta - v}{(w - v \cos \theta)^2} \right) = 0 \]
   \[ w \cos \theta - v = 0 \]
   \[ \theta_{\text{max}} = \cos^{-1} \left( \frac{v}{w} \right) \]

4. b) Values give \( \theta_{\text{max}} = 66.4^\circ \)
   Jogger at 90\(^\circ\) clears (\( L = 2 m \) = Hit)
   Jogger at 66.4\(^\circ\) clears 1.31m \( \Rightarrow \) Avoid

4. Ideals are
   - Trucker decelerates
   - Jogger accelerates
   - Jogger runs in a curve
   - Trucker and Jogger reaction times

\[ V_{\text{Ty}} = 0.020 \]
Mechanics Classwork 2: Gravitation

1. a) Assuming
   - Earth behaves as a point mass
   - No influence from external masses

   \( g = \frac{-GM_e}{r^2} \)

   \( g_h = \frac{-GM_e}{(R_e + h)^2} \)

2. \( g = 9.813 \text{ N kg}^{-1} \)
   \( g_h = 9.813 \frac{(3.557 + 14.174)^2}{(28.957 + 14.174)^2} \)
   \( g_h = 3.68 \text{ N kg}^{-1} \)

   A tiny fraction of \( g \).

3. \( \Delta g = \frac{-GM}{r^2} \times \left( \frac{1}{r^2} - \frac{1}{(r + h)^2} \right) \)
   \( = \frac{GM}{r^2} \left( 1 - \frac{1}{1 + \frac{h}{r_e}} \right)^2 \)
   \( \Delta g = \frac{GM}{r^2} \left( 1 - \frac{1}{1 + \frac{h}{r_e}} \right)^2 \)

   From the binomial theorem, 
   \( (1 + \frac{h}{r_e})^{-2} = 1 - 2 \frac{h}{r_e} + \ldots \)

4. As \( h \ll R_e \) this gives
   \( \Delta g \approx \frac{GM}{r^2} \left( 1 - 1 + \frac{2h}{r_e} \right) \)
   \( \Delta g \approx 2 \frac{GM}{r^2} \frac{h}{r_e^3} \)

   as required.

   Inputting numbers gives
   \( \Delta g = 3.08 \text{ N kg}^{-1} \)

   as before.

5. a) Object will accelerate towards Earth with increasing acceleration \( \frac{1}{x^2} \) until it collides with the surface. As \( M_e \gg m \) the Earth will barely move.

   b) Using \( F = ma \)
   \( \mu V dV = -GM_e m \frac{dv}{dt} \)

   So
   \( \frac{dV}{dt} = -GM_e \frac{t}{x^2} \)

   c) Integrating
   \( V = \frac{GM_e}{2} \frac{t}{x^2} \)

   \( \Rightarrow \frac{1}{2} V^2 = -GM_e \left( \frac{1}{x^2} + \frac{1}{x_0^2} \right) \)

   \( \Rightarrow V = \sqrt{2GM_e \left( \frac{1}{x_0^2} - \frac{1}{x^2} \right)} \)

   Increase in kinetic energy \( (\frac{1}{2} mV^2) \)

   = Loss of gravitational PE
   \( (-GM_e \frac{t}{x^2} + GM_e \frac{t}{x_0^2} \text{ gives the same result (and is probably simpler!}) \)

6. With \( z_0 = R_e + h \) and \( z = R_e \) then
   \( V^2 = \sqrt{2GM_e \left( \frac{1}{R_e} - \frac{1}{R_e + h} \right)} \)

   \( = \sqrt{2GM_e \left( R_e + h - R_e \right)} \frac{V}{R_e} \)

   \( \Rightarrow V = \sqrt{2GM_e \left( \frac{R_e + h}{R_e} \right)} \frac{V}{R_e} \)

   \( \frac{V^2}{2} = \frac{2}{R_e} G M_e h \), and as \( g = \frac{GM_e}{R_e^2} \)

   \( V^2 = 2gh \), which is the same as \( V^2 = \frac{W}{2} + 2\alpha \).

   e) For a point mass, \( z \to 0 \) so velocity would become infinite. Relativistic kinematics will apply.

   f) For the point mass, \( z \to 0 \) so the solution becomes
   \( \frac{t}{2} = \sqrt{\frac{2GM_e}{2z_0^3}} t \)

   \( \Rightarrow t = \frac{GM_e}{2z_0^3} \frac{t^2}{2} \)

   To get a rough estimate for rock collision time with \( z_0 = 1 \text{m} \), \( M_e = 1 \text{kg} \) gives \( t \approx 10^5 \text{s} \equiv 1 \text{day} \)
1. (a) Inside the ship astronauts perceive a downward acceleration field strength of \(3 \text{ ms}^{-2}\) on all objects:

Acceleration is constant so using \(s = ut + \frac{1}{2}at^2\) vertically gives the time taken to hit the ground by \(h = \frac{1}{2}a_0t^2\)

\[ t = \sqrt{\frac{2h}{a_0}} \]

Horizontally, ball moves at \(v_0\) so if it's just enough to just travel \(d\) then \(v_0 = d/t\) so \(d/v_0\)

Equating these times and rearranging for \(v_0\) gives a required velocity of

\[ v_0 = \sqrt{\frac{2dh}{a_0}} \]

So \(v_0 = 8.66 \text{ ms}^{-1}\)

(b) Inertial observer sees the ball moves straight across while the floor accelerates upwards at \(3 \text{ ms}^{-2}\)

Applying the same analysis but this time with the floor moving upwards gives the same result.

2. i) \(t = \frac{R}{v}\)

ii) Will have turned angle \(\alpha(t)\).

For straight line approximation this will be a distance of \(r(t) = R wt\)

iii) \(v = \frac{\Delta S}{r} = \frac{2v_0}{\sqrt{v_0^2 + r^2}}\) \(\frac{2v_0}{\sqrt{v_0^2 + r^2}}\) makes the \(S = \frac{2v_0^2}{\sqrt{v_0^2 + r^2}}\)

With \(\theta = 24.6^\circ\) \(\sin(24.6^\circ) = 0.41\)

\(5 = \frac{2 \times 5 \sqrt{v_0}}{\sqrt{v_0^2 + 5^2}}\)

With \(T = 14 \times 60 = 840 \text{ s}\), a 2m table, and \(v_0 = 8 \text{ ms}^{-1}\) (a slow shot) gives

\(S = 281 \text{ cm}\)

Baller might as if there is no real force on the ball. It has merely continued in a straight line as Newton's first law states.

b) World viewed from above with exaggeratedly large table on top.
The Slim Ladder Problem

The angular velocity of the Centre

Ladder is 2.6m and this is equivalent to the constant velocity of the mass of the ladder, which is 0.06 m/s. This results in the unbridled force being applied to the ladder.

The angle of attack is 0.05 radians and this is also equivalent to the mass of the ladder, which is 0.06 m/s.

The angular velocity of the Centre

Ladder is 2.6m and this is equivalent to the constant velocity of the mass of the ladder, which is 0.06 m/s. This results in the unbridled force being applied to the ladder.

The angle of attack is 0.05 radians and this is also equivalent to the mass of the ladder, which is 0.06 m/s.

The ladder is 2.6m and this is equivalent to the constant velocity of the mass of the ladder, which is 0.06 m/s. This results in the unbridled force being applied to the ladder.

The angle of attack is 0.05 radians and this is also equivalent to the mass of the ladder, which is 0.06 m/s.

The ladder is 2.6m and this is equivalent to the constant velocity of the mass of the ladder, which is 0.06 m/s. This results in the unbridled force being applied to the ladder.

The angle of attack is 0.05 radians and this is also equivalent to the mass of the ladder, which is 0.06 m/s.
In equilibrium:

\[ \frac{dS}{ds} = \frac{u}{c} \]

Forces drawn not quite aligned but they would be really.

\[ \text{Mass of block } = mghA \]

Mass of displaced fluid = \( d \times 5A \)

For equilibrium \( U = mg \)

\[ dS \Delta g = dS \Delta h \]

So \( dS = dS \Delta h \)

When pushed down, not in equilibrium, accelerates upwards.

\[ \Delta s \]

As is the displacement from equilibrium.

Applying Newton's 2nd Law:

\[ ma = U - mg \]

\[ m = \frac{dS}{s} \Delta h \]

and \( U = dF \Delta (s + \Delta s) \)

\[ dS \Delta a = dS \Delta (s + \Delta s) - dS \Delta h \]

\[ \Delta h = \frac{dS \Delta h}{s} + dS \Delta g + dS \Delta g - dS \Delta h \]

\[ a = \frac{dS \Delta g}{dS \Delta h} \]

\[ a = \frac{\frac{dS \Delta g}{dS \Delta h}}{\frac{1 + \Delta x}{s}} \]

from the figure:

\[ \frac{1}{2} = \Delta x + x \Rightarrow x = \frac{1}{2} - \Delta x \]

\[ \frac{1}{2} \Delta x = x \Delta x \]

\[ \frac{1}{2} \Delta x = x \Delta x \]

\[ \text{Giving} \]

\[ a = \mu g \left( \frac{x - \Delta x}{\Delta x - \Delta x} \right) \]

\[ a = \mu g \frac{2 \Delta x}{\Delta x} = \left( 2 \mu g \right) \Delta x \]

\[ \Rightarrow \text{Acceleration } \Delta x \text{ and acts to restore the plane to a central position.} \]

\[ \Rightarrow \text{Oscillates with angular frequency} \]

\[ \omega = \sqrt{\frac{2 \mu g}{s}} \]

\[ T = 2\pi \sqrt{\frac{d}{2 \mu g}} \]

\[ \Rightarrow \text{For SHM there is no loss of energy in a cycle. In this system energy is permanently being supplied by the rolling cylinders and lost as friction i.e. the system is permanently driven. The KE + PE is not constant.} \]

\[ \text{W. Symes Nov. 2010} \]
(i) A body’s rate of change of angular momentum is equal to the torque on the body (Newton’s 2nd Law in its rotational form) i.e. 
\[ \vec{\tau} = \frac{d\vec{L}}{dt} \]

(ii) The torque on a planet orbiting a star is zero as the force on the planet is along the same line as the star and 
\[ \vec{F} = \vec{F}_x = rF \sin \theta \]

\[ = 0 \text{ as } \theta = 0 \text{ hence } \frac{d\vec{L}}{dt} \text{ is zero so } \vec{\tau} \text{ is constant.} \]

(iii) A body’s rate of change of angular momentum is equal to the torque on the body (Newton’s 2nd Law in its rotational form) i.e. 
\[ \vec{\tau} = \frac{d\vec{L}}{dt} \]

\[ h = \vec{r} \times \vec{v} = \frac{1}{2} r^2 \Omega \text{ so area} \]
\[ SA = \frac{1}{2} rh = \frac{1}{2} r^2 \Omega t \]

\[ \Rightarrow \frac{SA}{St} = \frac{1}{2} r^2 \Omega \]

So as \( St \to 0 \)
\[ \frac{dA}{dt} = \frac{1}{2} r^2 \Omega \]

Now \( \vec{L} = \vec{r} \times \vec{F} \)
\[ \Rightarrow L = rF \sin \theta \]

So \( r \sin \theta = L/m \)

So \( \frac{dA}{dt} = \frac{1}{2} m \)

As \( m, L \) are constants \( \frac{dA}{dt} \) is constant.

(iv) By rearranging \( L = \sqrt{\frac{(GM)^2}{h+5}} \)

\[ T = \frac{2\pi}{T^2} = \frac{GM^2}{h+5} \]

\[ = 31,857,600 \text{ s} \]

\[ \Rightarrow \text{eccentricity } = 1.5 \times 10^{-11} \text{ m} \]

Circular orbits so planets go in opposite directions and maintain their distance.

For \( M_1 \), using the result from (iii)
\[ \frac{4\pi^2}{T^2} r_1 = G \frac{M_1}{(r_1 + r_2)^2} \]

And for \( M_2 \)
\[ \frac{4\pi^2}{T^2} r_2 = G \frac{M_2}{(r_1 + r_2)^2} \]

Adding (i) and (ii) gives
\[ \frac{4\pi^2}{T^2} (r_1 + r_2) = G \frac{M_1 (M_1 + M_2)}{(r_1 + r_2)^2} \]

Rearranging
\[ T = \frac{2\pi}{\sqrt{\frac{(r_1 + r_2)^3}{G (M_1 + M_2)^{3/2}}} \]

Wiggins Dec 2010
1. Before

\[ \begin{align*}
M & \quad \rightarrow \quad u \\
\Rightarrow & \quad \omega \\
\Rightarrow & \quad \omega \quad \downarrow \\
\Rightarrow & \quad \omega \quad \rightarrow
\end{align*} \]

After.

\[ \begin{align*}
M & \quad \rightarrow \quad u \\
\Rightarrow & \quad \omega \quad \downarrow \\
\Rightarrow & \quad \omega \quad \rightarrow
\end{align*} \]

Momentum before

\[ m \cdot u = m \cdot u' + M \cdot \omega \]

Momentum after

\[ m \cdot u = m \cdot u' + M \cdot \omega \]

Collision is elastic so kinetic energy is conserved, so that

\[ \frac{1}{2} m u^2 = \frac{1}{2} m u'^2 + \frac{1}{2} M \omega^2 \]

\[ u^2 = u'^2 + \omega^2 \]  

From (1), \( u^2 \) can be written

\[ (v^2 + \omega^2)^2 = v^4 + \omega^4 + 2v^2 \omega^2 \]

From (2) this gives

\[ v^2 + \omega^2 = v'^2 + \omega'^2 \]

Hence \( \cos \theta = 0 \)

\[ \Rightarrow \theta = \pi/2 \] and as required.

\[ \boxed{\text{N.B. if inelastic then} \quad u'^2 = u^2 + 2\Delta E/m, \quad \text{where } \Delta E \text{ is the lost energy and this would give} \quad 2\omega \cos \theta = 2 \Delta E/m, \quad \text{so} \quad \theta = \cos^{-1}(\Delta E/m)} \]

2. a) As \( M \) sticks to the dumbbell by the Conservation of Linear Momentum

\[ m \cdot u = (2M + m) \cdot v \]

So \( v = \left( \frac{m}{2M+m} \right) u \]

Which gives \( u = \frac{v}{\frac{2}{3}} = 0.8 \text{ m/s} \)

b) Can take the Conservation of Angular Momentum, easiest to do this about the centre of mass which has vertical coordinate

\[ \Rightarrow y = \left( \frac{M}{m+2M} \right) l \quad (0.8 \text{ m}) \]

Angular momentum before collision = \( \frac{2}{3} \cdot \theta \)

\[ = y \cdot m \cdot u \quad \text{out of the paper} \]

\[ = \left( \frac{M}{m+2M} \right) l \cdot u \quad (1.6 \text{ Nm}) \]

\[ \text{and is constant.} \]

The angular velocity after the collision is needed so we can say

\[ I \cdot \omega = \left( \frac{M}{m+2M} \right) l \cdot u \quad \text{where} \]

\[ I \quad \text{is the moment of inertia about the centre of mass.} \]

\[ I = \sum m \cdot r^2 \quad \text{gives} \]

\[ I = \frac{M(m^2+4m+8)}{2M+3m} \quad (2.4 \text{ kg m}^2) \]

Putting this back into the previous angular momentum equation yields,

\[ \omega = \left( \frac{m}{M+m} \right) \frac{u}{l} \]

Which gives \( \omega = \frac{2}{3} \cdot 0.66 \text{ rad/s} \)

c) KE before = \( \frac{1}{2} m u^2 \)

KE after = \( \frac{1}{2} m v'^2 + \frac{1}{2} I \omega^2 \)

Which becomes after some algebra

\[ \text{KE after} = \frac{1}{2} \cdot \frac{m^2}{M+m} \omega^2 \]

Giving the loss in KE as

\[ 1 m u^2 - \frac{1}{2} \cdot \frac{m^2}{M+m} \omega^2 \]

\[ \approx \frac{1}{2} M \frac{m}{M+m} u^2 \]

Which gives

\[ \text{KE lost} = 8/3 \cdot 2 \cdot 6.75 \]

Lost energy is dissipated as heat, maybe an audible click is solid spheres colliding.

V. Tynns Dec 2010