Complex Analysis - Vvedensky

- The Cauchy Product $\left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} a_k b_{n-k}\right)$

- $\left.\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned}\right\} \therefore \cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}, \quad \sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$

- $\cos z = \cos x \cosh y - i \sin x \sinh y$
  $\sin z = \sin x \cosh y + i \cos x \sinh y$

- $\ln z = \ln(re^{i\theta}) = \ln r + \ln e^{i\theta} = \ln r + i\theta$
- $z^a = e^{a \ln z}$

For $b^2 - 4ac > 0$; $\quad y(x) = Ae^{m_1 x} + Be^{m_2 x}$

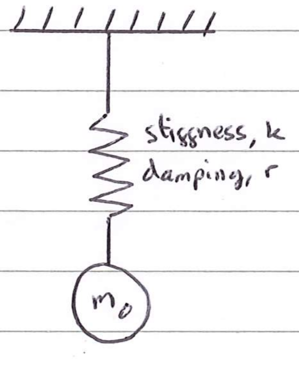
For $b^2 - 4ac = 0$; $\quad y(x) = (A + Bx)e^{mx}$

For $b^2 - 4ac < 0$; $\quad y(x) = Ae^{mx} + Be^{m^* x}$

$$= e^{\alpha x}(C \cos \beta x + D \sin \beta x)$$

(Re, Im)

The Harmonic Oscillator: (About the equilibrium)



"$F = ma$" ($\uparrow$):

$$m_0 \frac{d^2 x}{dt^2} = -k\left(x - \frac{m_0 g}{k}\right) - r\frac{dx}{dt} - m_0 g$$

$$\therefore \ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

natural frequency, $\omega_0 = \sqrt{\frac{k}{m_0}}$

damping factor, $\gamma = \frac{r}{m_0}$

$$\Rightarrow m = \tfrac{1}{2}\left(-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}\right)$$

For $\gamma^2 - 4\omega_0^2 > 0$; overdamped 

For $\gamma^2 - 4\omega_0^2 = 0$; critically damped 

For $\gamma^2 - 4\omega_0^2 < 0$; underdamped